ELASTIC CYLINDRICAL SHELLS WITH OPEN AND CLOSED ENDS

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Abstract

LINE LOAD ON CYLINDRICAL SHELL AND END PLATES

Part I

The elastic static stresses and displacements in various shells $\left(a/\ell = 1/2;\ a/h = 50,75,100\right)$ with end plates $\left(h/h_p = 1/2,1,2\right)$ are determined when the cylinder is subjected to a uniform line load along a generator. The method of solution is the superposition of a particular solution in which the shell's superposition of a particular solution in which the shell's response is computed for a simply supported end conditions (no axial moment, axial force, radial displacement or circumferential displacement) and a complementary solution in which the true end plate reactions are imposed on the ends of the cylinder. Examination of the end conditions reveals that the assumptions N_X = w = 0 (no axial force and radial displacement) are generally justified whereas the assumptions M_X = V = 0 (no axial moment and circumferential displacement) will usually lead to considerable relative errors in the axial bending stresses and the circumferential membrane stresses, respectively. Experimental results are in good agreement (generally within ~10%) with the computed solutions.

RADIAL DEFORMATION IN THE BOUNDARY VALUE PROBLEM OF THE MULTIPLE TIER CYLINDRICAL SHELL

Part II

Hoff's solution [4] of the homogeneous Donnell equations is used to study the radial deformation due to non-symmetric axial settling in vertical multiple tier cylindrical storage

tanks with a stiffener ring at the top. The solution of a uniform thickness shell problem is shown (by example) to approximate a "safe" displacement solution to the multiple tier problem if the minimum multiple tier thickness is used for the constant thickness solution. Although axial slopes decay rapidly along the cylinder for all cases, the resulting radial displacements may be significant throughout the shell. Hoff's analysis is compared with the inextensional theory for very thin shells ($a/h = 100$ to 1500; $\ell/a = 0.1$ to 3) and for various harmonic numbers ($n = 2$ to 15). The accuracy of both the uniform thickness and the inextensional approximations generally increases with increasing a/h, decreasing ℓ/a, and decreasing n. Energy considerations are shown to infer the general character of a high agreement (95%) region between the Hoff solution and the inextensional analysis. By studying the deformation in existing storage tanks with a stiffener ring at the top (e.g., $a \approx 1100$; $h_i \approx 1.25$ to 0.5; $\ell_i \approx$ where i = number of tiers = 8; stiffener ring is 3 x 3 x ¼ standard angle; material : steel) it has been found that the ring is of little value (max. reduction in radial displ. $\approx 7\%$ at $n = 15$).