

# CONSTITUTIVE RELATIONS FOR FINITE ELEMENT ANALYSIS OF TENSION STIFFENING IN REINFORCED CONCRETE

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## **Abstract**

In recent years, there has been growing interest in finite element analysis of concrete structures and at the same time an increasing awareness of some of the difficulties encountered in these calculations. This dissertation concentrates on such difficulties and seeks to suggest improvements on aspects of computing efficiency, accuracy and reliability.

The main feature of a finite element program for static analysis of reinforced concrete plane frames is described. In contrast to the conventional multi-layering technique, a *two-layer* approach (using the compression zone and the tension zone) to allow the nonlinear material properties is proposed. Lengthy integration through the thickness in the former is replaced by simple interpolation on the constitutive relations stored in a special form, leading to direct computation of internal force and moment according to the concrete compressive extreme fibre strain and the tensile steel strain. This method is useful and should result in a more efficient nonlinear solution algorithm.

Emphasis is placed on the reliability of the solution procedure, both in the service ability and ultimate load ranges. The arc-length method is re-formulated for passing limit points. In the method, the out-of-balance load is resolved into a parallel and an orthogonal component with respect to the direction of the external applied loads. The selection of roots and the occurrence of complex roots to the quadratic equation of the arc-length constraint are discussed. The method is effective and versatile in handling both snap-through and snap-back problems.

Experiments to measure the constitutive relations for tension stiffening are described in detail. Eccentric tension tests were conducted on reinforced concrete beam specimens, with the neutral axis maintained at a position close to the compression face throughout the tests hence the tension zone properties could be studied directly. The total tensile force and its position were expressed in a form suitable for direct input to a computer.

A procedure involving interpolation on the experimental results in finite element calculations is developed. This allows for changing tension zone parameters as the neutral axis position changes hence a more realistic analysis can be carried out. A strategy to allow for material unloading is also detailed.

Examples of analysis using the finite element program are given. Comparison of the results with those from other sources shows satisfactory agreement. The proposed methods are useful for a wide range of concrete plane frame problems.