Folded Shell Structures

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Abstract

A novel type of shell structure was analysed, *folded shell structures.* These shell structures have a distinct structural hierarchy: globally they can be regarded as thin-walled shells, but at a meso scale they consist of tessellated unit cells, which in turn are composed of thin-walled shells joined at distinct fold lines. It is this structural hierarchy that imbues the folded shell structures with their interesting mechanical properties. The global sheet deformations are a combination of bending along the folds, and deformation of the interlying material. The former is primarily a kinematic problem, with a parallel in the flexibility of hinged plate structures. A review of the mathematics of rigid origami provides the necessary background to develop non-trivial geometries of these folded shells that still exhibit a soft deformation mode.

Two example folded shell structures are introduced, the Miura and Eggbox sheet. Both consist of a tessellation of parallelogram facets; the first is developable, while the other has points of positive and negative Gaussian curvature. The first property of interest is their increased in-plane flexibility, by virtue of the opening and closing of folds. The Miura and Eggbox sheet respectively have an effective negative and positive in-plane Poisson's ratio. Secondly, both sheets can modify their global Gaussian curvature, with no stretching at the material level. Thirdly, both sheets exhibit an oppositely signed Poisson's ratio for in-plane and out-of-plane deformations; *e.g.* when bending the Miura sheet it exhibits a negative Poisson's ratio behaviour and deforms anticlastically.

The salient global deformations of the sheets were analysed in terms of the kinematics of the constituent unit cells. The characteristic in-plane and out-of-plane properties of the sheets followed directly from developable deformations of the tessellated unit cells. A more holistic top-down numerical approach modelled the sheets as an array of unit cells. The sheets were represented by a pin-jointed bar framework, and additional planarity constraints between facets enabled the inclusion of a bending stiffness for the facets and fold lines. A modal analysis of the sheet's stiffness matrix showed that the characteristic deformation modes are among the dominant eigenmodes of the sheets for a wide range of geometries and material properties.

Many folded shell structures can be folded from flat sheet material, with only minimal material deformations. The manufacturing processes must overcome the intrinsic kinematics of the sheets, whereby the sheet contracts in two directions simultaneously. Existing methods were reviewed, and classified into synchronous folding, gradual folding and pre-gathering techniques. A novel cold gas pressure manufacturing method was introduced, and it was shown that a simple plastic hinge model cannot yet fully account for the total required forming energy.