Abstract

A major obstacle for the realisation of higher-order accuracy for immersed or embedded domain finite element methods is the problem of numerical integration in cut-cells with complex shape. We propose a new method to recover surface and volume integration domains defined by implicit geometry. First, we generate a linear reconstruction of the implicit geometry using template-based procedures. Next, a higher-order tessellation is constructed by introducing additional nodes and placing them on the immersed geometry. The use of implicit geometry and locality of interrogation makes the intersection computations used in this step highly robust. The algorithm is further augmented with a facility to preserve sharp features and specially-designed refinement routines, which ensure cut-cell topology is restricted to a small set of possibilities.

The implicit geometry can be obtained directly or constructed from a wide variety of input data types by leveraging a rich theory developed by the computer graphics community. The construction step is frequently inexpensive and in many cases, including some spline-based surfaces, purely algebraic and exact. Additional enhancements to conventional engineering workflows, such as robust handling of poor quality CAD input and filtering of small geometric details, which can be difficult to implement for methods based directly on parametric geometry, are almost automatic.

Although our method is implemented in an existing framework using tensor product b-splines defined on a Cartesian background grid, this is not a principle restriction; the method also applies to general unstructured embedding meshes. Furthermore, the method does not require complicated data-structures and generates integration meshes local to each background cell, independently of all neighbours. The latter property is highly desirable in our case since integration meshes are automatically aligned with the knot-spans of the background splines thus side-stepping additional complexity present in other approaches.