THERMAL BUCKLING OF CIRCULAR PLATES

B Cotterell

Abstract

Circular plates are considered in their post buckled condition for:

i) Symmetrical heating over a central area.

ii) Uniform heating round the circumference.

In both cases only the steady state is considered. The normal methods of approximate analysis were unsuitable for solving the non linear equations. A digital computer was used.

The deflection of a centrally heated plate is saucer like with radial symmetry. The differential equations were broken down to the first order and integrated using the Runge-Kutta-Gill method, which gave 1% accuracy when 50 steps were taken. There were two unknown centre variables caused by the two point boundary conditions. Theses had a multiplicity of roots due to higher modes and only separate linear interpolation of each variable, satisfying first the radial stress and then the bending moment at the edge, gave convergence. Edge restraints did not greatly affect either the shape or the magnitude of the buckle. There were two reasons: a circular plate lacks stiffness in radial bending; the mean stresses were affected only slightly, because the radial displacements were small.

The first mode of buckling of a plate heated round the circumference is saddle-like and expressible as the sine series:

$$W = \sum_{m=2,6,10\ldots}^{\infty} \rho_m(r) \sin m\theta \,,$$

By taking two terms it was shown that, for deflections up to six times the thickness, the first gives 1% accuracy. The differential equations were integrated as previously. The unknown centre variables were found by the Newton Raphson process. The interpolation was easier than before because higher modes could not be present. The critical temperature is less than

for a centrally heated plate, but the buckle is more easily restricted. A large edge member, because it is hot, can stretch the plate and prevent buckling. Also, since the edge of the plate is not in one plane, perpendicular restriction has a large effect.

A centrally heated free plate was investigated experimentally. Infra-red lamps provided the heat source. Temperatures and strains, were measured with thermocouples and resistance gauges. Since self compensated gauges were not available, each gauge was calibrated. Because the calibration formed a large percentage of the total readings, the accuracy was lowered. Although the experimental mean stresses were unreliable, the bending stresses and deflections were reasonably consistent with theory.