NUMERICAL SOLUTION OF SOME PLATE AND SHELL PROBLEMS, WITH EMPHASIS ON COLLOCATION

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Abstract

A procedure is developed in which algebraic polynomials are used to find approximate analytical solutions for linear elastic equilibrium problems in two-dimensional continua.

A solution is assumed in the form

$$w = F(x,y) \sum_{m=o}^{L} \sum_{n=o}^{L} A_{mn} x^m y^n$$

where A_{mn} are arbitrary coefficients and F(x,y) is a known polynomial in x and y by which the assumed solution may satisfy all, some or none of the boundary conditions. In the last case F(x,y) = 1. The undetermined coefficients are found by means of simple methods such as Collocation or Subdomain.

An important feature of the approach presented is that, unlike the classical methods of weighted residuals, the approximating functions do not need to satisfy all the boundary conditions and they are always assumed as a square array of simple truncated power series regardless of the complexity of the boundary conditions. This feature reduces the arbitrary decisions of the user and makes the method suitable for automatic computing machines.

Series of Legendre polynomials are considered for plate problems and are compared with solutions obtained by means of simple power series. The power series are found to be, by virtue of their simplicity, the most suitable approximating functions.

Various problems of torsion, thin plates under transverse load and circular cylindrical shells are chosen to illustrate the application of the method, and comparison is made with other available answers. The problems include rectangular, triangular and rhomboid shapes with clamped and free edges.

In all the problems, the effect of collocation in difference patterns is studied. The convergence is shown according to the number of equations used to determine the arbitrary coefficients and the influence of acute and obtuse corners on the convergence is also studied.

Finally, the principal bending moments and their contours, when Poisson’s ratio is zero, are presented for the plate problems. This has been done in order to provide data for at type of optimum design of concrete slabs based on elastic analysis which has been found to be an economic way of dealing with this type of structures. The quantity of interest is expressed as a “moment-volume” and is computed for various plates solved.