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Novel Symmetric Tensegrity Structures

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by

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This dissertation investigates novel forms of tensegrity structures, using *stress matrices* written in a symmetry-adapted coordinate system to simplify the structural analysis. *Tensegrity structures* are made rigid by a state of *self-stress* or *prestress*. A challenging step in the design of these structures is the determination of their self-equilibrated configurations, known as *form-finding*.

The key to the stability of tensegrity structures is their *geometric* stiffness. The geometric stiffness can be described using the *stress matrix*, which is written in terms of the *tension coefficients* for each member of the structure. For three-dimensional structures, if the stress matrix has a nullity of four, and positive semi-definite eigenvalues, then the structure is said to be *super-stable*, meaning that the structure is guaranteed to be unconditionally stable, without relying on the material stiffness. However, for a typical tensegrity structure, the stress matrix will be large and unwieldy. Using symmetry can make the matrix more tractable: group representation theory can be used to find symmetry-adapted coordinate systems for the structure. The analysis of the structure can then be simplified by using these symmetry-adapted coordinate systems to block-diagonalise matrices which describe the structure's behaviour. In particular, the block-diagonalised stress matrix provides valuable insight into finding a set of tension coefficients that achieve equilibrium configurations for three-dimensional tensegrity structures. *Translational symmetry* along with point group symmetry is also applied to simplify the analysis of *repetitive tensegrity structures*.

In the present study, we show how to generate a catalogue of stable tensegrity structures, using point group theory. First, we choose the point group symmetry of the tensegrities that we are trying to find: here they may have *icosahedral*, *cubic*, or *tetrahedral symmetry*. We assume that there is a *regular orbit* of nodes (each can be considered as corresponding to one symmetry operation), one orbit of struts, and two orbits of cables. For each possible choice of cables and struts, we look for equilibrium configurations by ensuring that the stress matrix is rank deficient by at least 4. It turns out to be straightforward to do this in terms of the known 3×3 *irreducible representation matrices* that correspond, for each symmetry operation, to the transformations of the x, y and z coordinates of a node. Once the equilibrium configuration has been found, it can then be checked for stability and accepted if it is a stable configuration, or rejected otherwise. We also generate a catalogue of *simple tensegrity structures* which will have a single, but not regular, orbit of nodes, one orbit of struts, and one orbit of cables, using the concept of site symmetry. It turns out that many choices of cables and struts give stable tensegrity structures.